June 28, 2006 Time: 90 min.

## Calculators and Mobile Phones are not allowed.

1. Evaluate each of the following limits, if it exists:

a) 
$$\lim_{x \to 1} \frac{x - 1}{\sqrt{2x} - \sqrt{3 - x}}$$

(3 Points)

b) 
$$\lim_{x\to 0} \frac{\sin x}{\sqrt{x^3+x^2}}.$$

(3 Points)

2. Use the 
$$\epsilon$$
,  $\delta$  definition of limit to show that:  $\lim_{x\to 2} (5-2x) = 1$ .

(3 Points)

3. Let

$$f(x) = \frac{(x^2 - 2x + 1)|x|}{(x^2 - 1)x}.$$

a) Find all points of dicontinuity of f and classify each discontinuity as removable, infinite, or a jump.

(3 Points)

b) Find the vertical and horizontal asymptotes for the graph of f, (if any).

(3 Points)

4. Show that the equation  $x^5 + 2x - 5 = 0$  has at least one real root.

(3 Points)

5. Assume  $\lim_{x\to -1} f(x)$  exists and

$$\frac{x^2+x-2}{x+3} \leq \frac{f(x)}{(x-1)^2} \leq \frac{x^2+2x-1}{x+3}.$$

Find  $\lim_{x\to -1} f(x)$ .

(4 Points)

6. Let 
$$f(x) = (x^2 - 1) \sec(\sqrt{x - 1}) + \tan(x^3 - 1)$$
. Find  $f'(x)$ 

(3 Points)